We publish below the Presidential Address of the Indian Academy of Sciences delivered by R. Narasimha at the 58th annual meeting of the Academy at Ahmedabad. It will be our endeavour to publish the Presidential Addresses in the 10th January issue each year.

—Editor

Turbulence on computers*

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Turbulence in fluid flows remains an unsolved problem, at once of great technological relevance and scientific interest, on which every available tool—theoretical, numerical, experimental—has been used. With the advent of computers engineers have invented many ad hoc mathematical models, and used them with effect in limited classes of flows. Direct ("exact") numerical solution of the governing equations can offer great insight at modest Reynolds numbers, and has just now become possible in India with the availability of a powerful parallel computer at the National Aeronautical Laboratory, Bangalore. An assessment of the various ways in which computer power can be harnessed for tackling the problem of turbulent flows is presented.

THE understanding, prediction and management of turbulent flows continue to present the greatest challenge to fluid dynamicists, and indeed to physicists and mathematicians as well. The scientific study of the subject may be said to have begun with the celebrated paper of Osborne Reynolds[1] in 1883. Its first sentence, which reads 'The results of this investigation have both a practical and philosophical aspect', already points to that combination of great technological relevance and deep physics and mathematics that explains the enduring fascination of the subject to several generations of scientists and engineers. Reynolds, born exactly a hundred and fifty years ago, was a professor of engineering at Manchester at a time when Victorian England was at the height of its political and industrial power, and was a pioneer in what we would today call engineering science[2]. Reporting experiments on the flow of water in a pipe, he showed (by observation of a filament of dye introduced into the pipe, and by measurements of pressure drop along the pipe for given flow rates) that there are two possible states of motion depending on the flow velocity: one smooth and regular (now generally called laminar), and another irregular and chaotic (called 'sinuous' by Reynolds, and 'turbulent' later by Kelvin[3]—a term that is now universally accepted). The transition from laminar to turbulent flow was later (1895) shown by Reynolds[4] to occur when a non-dimensional group, since called the Reynolds number following Arnold Sommerfeld's proposal[5] in 1908, exceeds a critical value; this group is defined as $\mathrm{Re} = VD/\nu$, where V is a characteristic flow velocity, D the diameter of the pipe and ν the kinematic viscosity. Usually the critical Reynolds number (based on average flow velocity) is quoted as 2300, but values higher by a factor of a hundred have been reported when great care is taken in conducting the experiment—there may actually be no upper limit in general[6].

A similar transition occurs in every known shear flow as its Reynolds number is increased: at sufficiently high Reynolds numbers all flows tend to be turbulent.

Now in spite of more than a century of effort, using the most powerful experimental, theoretical and computational tools available, turbulent flow remains ununderstood: Feynman called it the greatest puzzle of classical physics, but the evidence is growing that it is not a mere puzzle, and may need new physics and mathematics, as von Neumann[7] foresaw in a remarkably perceptive essay written for the US Office of Naval Research in 1949. In his well-known treatise on hydrodynamics Horace Lamb[8] began a brief discussion of turbulent motion with the words, 'It remains to call attention to the chief outstanding difficulty of the subject'. He is also reported to have said, at the British Association meeting of 1932 (ref. 9), '... when I die and go to Heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics, and the other is the turbulent motion of fluids. And about the former I am really rather optimistic'.

Of course a great deal is *known* about turbulence, but whatever is so known has to appeal to (if indeed it does not critically depend upon) test data of some kind or

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other: the information required from experiment may by clever analysis be reduced (usually by simple applications of a certain kind of group theory[10], involving the assumption of some kind of similarity among the solutions), but it cannot be eliminated. Liepmann[11] has charted 'the rise and fall of ideas' in turbulence, and shown how the history of the subject is littered with the debris of 'promising' ideas that have rarely lived up to their promise.

This is an unsatisfactory state of affairs for two reasons. First of all the basic laws governing the motion of fluids, including in particular when in a state of turbulent flow, are known beyond reasonable doubt; the laws are contained in the Navier–Stokes equations, written down for the first time by the French engineer Navier in 1822 (ref. 3). Because of their strong nonlinearity, however, no solutions of these equations relevant to turbulent flow are known, not even for the simplest geometry. Computer solutions are only just now being obtained, fully one and a half centuries after Navier declared that 'The true engineer always calculates'.

Secondly, while a great deal *is* known based on experimental information, the lack of a fundamental *understanding* of the nature of the solutions means that certain classes of questions cannot be answered, perhaps not even raised. For example, during the last two decades several novel methods of turbulence management (which would include both promotion and suppression, depending on the application: promotion could increase heat transfer, suppression could reduce drag) have been studied, and some have been quite successful[12]. Thus, it is now established beyond doubt that fine streamwise grooves on a surface, forming what are known as 'riblets' (height and width of the order of a millimetre in aircraft applications), can reduce turbulent skin friction drag by 5–10%. (A recent victory of the United States in the Americas Cup has been attributed, although not entirely convincingly, to the use of ribbed sails. It would not be surprising if aircraft surfaces in the next decade are rough to the touch, the engineered roughness resulting in lower fuel consumption!) Now the point is that such turbulence 'managers' derive little inspiration from any theory available today: they have come rather from a sophisticated kind of tinkering with the flow by engineers steeped in experimental lore, and in particular by the discovery that there is much order ('coherent structure') in the apparent chaos of turbulent flow[13,14].

There is nothing more practical than a good theory, but such a theory does not exist for turbulence yet. In this situation, three lines of attack on the subject have emerged: Figure 1 summarizes the relations between the approaches that have been adopted. First, experimentation, and there has been a great deal of this which now needs more sophisticated 'cataloging' than has been done till now, along the lines of Coles's *Young Person's Guide to the Data* on boundary layers. Secondly (and here the computer makes its first entry), one can construct mathematical models that engineers could solve for making estimates in design. This has become an industry in its own right now, and computer codes of various kinds have proliferated in the market place[16]. Many of these are refinements of ideas put forward by Ludwig Prandtl[17], the founder of modern engineering fluid dynamics. Prandtl was, at one end of the spectrum of such models, the author of the simplest of them, which proposes a mixing length analogous to the mean free path of the kinetic theory of gases — but unfortunately the analogy is not strictly valid, as in turbulent flow eddy interaction and separation distances are of the same order, making the statistical-mechanical situation closer to that in a liquid rather than a gas. (Eddies experience not the rare but sharp encounters of the balls on a billiard-table, but the continuous jostling of people in a crowd.) Prandtl[18] also suggested, at the other extreme, a rather complicated partial differential equation to compute a velocity scale for any turbulent flow. Kolmogorov[19] and Rotta[20] made proposals along similar lines, and these have been pursued vigorously with various refinements ever since.

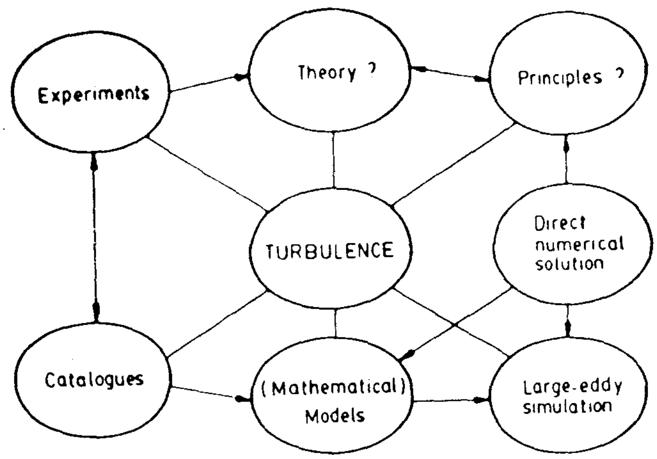

Figure 1. ***Different approaches to turbulence.*** Mathematical models (currently all *ad hoc*) have till now been chiefly derived from experimental data, but may in future be influenced by direct numerical solutions of the Navier-Stokes equations, which are best seen as providing data from numerical (rather than physical) experiments. Large-eddy simulation is a hybrid, with modelled small-scale motion. Experiments and DNS should help discover principles and construct or verify theories, of which there is nothing satisfactory today.

By tuning the empirical parameters in such models one can obtain reasonable agreement with experimental data in certain classes of technologically important flows, and it has even been possible to produce, for example, novel aerofoils that give enormously high lift-to-drag ratios under certain conditions[21]. But these models have remained models and no more: the kind of luck that Planck had, when his quantum 'model' for radiation turned out actually to hit the 'truth', has not favoured fluid dynamicists[16]. In a way the turbulence models now in wide use in industry are very sophisticated 'interpolations' among experimental data — interpolations carried out through the medium of partial differential equations, so to speak — to be used with caution outside the range of validation or the class of flows experimentally investigated. If around the turn of the century hydraulics depended on empirical algebraic relations (dismissively referred to by the great aeronautical fluid dynamicist von Karman as the 'science of variable constants'), in the age of computers we have empirical partial differential equations. The scientific basis of these models is weak, in the sense that their connection with the Navier–Stokes equations is dubious. But they have served the purposes of engineering design within a carefully limited area: as Heaviside pointed out, one does not wait to understand the process of digestion before starting to eat. It is important with such models to appreciate what they can and cannot do, to exploit them where possible, to treat them with caution and scepticism where necessary[16].

The third possibility is to compute numerically exact solutions of the full, unsteady, three-dimensional Navier–Stokes equations. (Even if the overall geometry of the *mean* flow is simple and possesses symmetries, e.g. axisymmetry in a round jet, real-life turbulent flow is basically three-dimensional in the fluctuations, so all components of the velocity vector need to be retained.) Right from the early days of electronic computers turbulence has been near the top of the computing agenda: in that same ONR essay von Neumann[7] spoke of 'some hope to "break the deadlock" by extensive, but well-planned, computational efforts'. It has long been known that this is very demanding on computer power: in 1970 Emmons[22] concluded that 'like every previous attempt at solution this one fails too', but, noting the rapid pace at which computer speeds had been growing, suggested that 'in the not-too-distant future the low Reynolds number turbulent flows may be within range of investigation'. In actual fact the first such solutions were published only two years later[23].

The problem in obtaining direct Navier–Stokes solutions (often abbreviated to DNS, also standing for direct numerical simulation — i.e. without the use of *any ad hoc* modelling) is the vast range of time and space scales that need resolution. The largest scales in a turbulent flow are comparable to the scale of variation of the mean flow (say diameter of pipe or jet); the smallest are those where viscous action is strong, and are named after Kolmogorov, who first characterized them in terms of the dissipation of turbulent kinetic energy to heat[24]. The ratio of the large eddy scale to the Kolmogorov scale increases with Reynolds number like $Re^{3/4}$, so volumes corresponding to these scales go like $Re^{9/4}$; time scales go like $Re^{1/2}$. Based on considerations like these it can be shown[25] that the total number of operations to compute even the highly idealized case of *homogeneous* turbulence (i.e. a flow that is statistically homogeneous in space) is proportional to

$$Re^{11/4}\,[c_1 \log_2 R^{3/4} + c_2],$$

where c_1 and c_2 are constants; and the storage requirement is proportional to $Re^{9/4}$. The time required for computing one realization of homogeneous turbulence on a 100 MFLOPS computer works out to 3 min at $Re = 100$, and 9 h at $Re = 400$ (these Reynolds numbers being based on large-eddy length and velocity scales). The computation of *high* Reynolds number turbulence must therefore await vastly more powerful computers before becoming practical, but relatively low Re flows are amenable to supercomputers of the present day.

This kind of computing power has not been available in India till recently. But with the commissioning of the Flosolver Mk-3 at NAL[26], the required computing power is now on hand, and the first such solution has been reported by Basu *et al.*[27]. As this has appeared in print recently in *Current Science*, I shall not devote space here to describe it again.

The great advantage of DNS is that it provides access to *any* flow variable, including all those that are virtually impossible to measure at present; e.g. the three-dimensional chaotic vorticity field that is the defining characteristic of real-life turbulence, or the total viscous dissipation to heat that governs the dynamics of both large and small eddies. It is therefore possible to use DNS to obtain insights that are not available from experimental techniques[27]. Of course DNS has its limitations as well: apart from the fact that it can now handle flow past bodies only at Reynolds numbers of the order of 10^3–10^4 (based on body size and free-stream velocity), there are (somewhat surprisingly) difficulties associated with prescribing boundary conditions on the surface of the (necessarily finite) computational domain within which the equations are solved. It does sometimes happen therefore that different simulations of turbulence for the same flow problem do not agree among themselves, a situation that is strangely similar to the difficulties that wind tunnel engineers have in accounting for what they call 'wall interference'!

As computers grow in power the Reynolds numbers at which DNS can be carried out will also increase, but are unlikely to match the values encountered in aerospace technology (where they are of order 10^7–10^8) atleast for another decade. But, quite apart from this consideration, there is a further paradoxical difficulty with DNS, namely that, for the vast majority of applications, it provides *too much* information. It is not uncommon for a new problem to take a year or two in formulation, execution and verification: but the analysis of the results may continue for several years. It is becoming the standard practice to create an archival database containing carefully verified computer solutions of the Navier–Stokes equations, to be retrieved and analysed against any new idea or proposal that may emerge in the course of research. But to obtain an immediate answer to an engineering problem direct numerical simulations will turn out not only to be numerically and financially expensive, but in fact to generate far more information than is necessary: DNS may be too 'rich'.

There is therefore an urgent need in applications for something that is more rational than the *ad hoc* models in current engineering use but less expensive and rich than DNS. This hybrid could well turn out to be the technique known as large eddy simulation (LES)[28,29]. The basic idea here is that the large scale motions in the flow, which are known to be flow-specific and sensitive to initial conditions (they have a long memory[30]), should be solved 'exactly' on the computer except for their interaction with the small scales. The small scales, which are much more nearly universal than the large scales (from which they receive their energy, which is then dissipated into heat as in Richardson's celebrated jingle[31]), are handled by a suitable model rather than by exact numerical solution of the Navier–Stokes equations. This procedure, which is basically what is widely used in numerical weather prediction (weather being a complex form of turbulence on a gigantic scale), has the advantage that dynamics is directly invoked only where it is essential, and is therefore both financially and intellectually economical. Its success however depends crucially on the ability to formulate adequate universal models for the fine structure of turbulence. This has been the subject of much fundamental research since the time of Kolmogorov's profoundly influential papers[24]. Although the last word on the subject has not yet been said, enough progress has been made in recent years to give cause for hope[32].

There is one other alternative line of attack. As turbulence is chaotic vorticity, and vorticity is a more compact field than the fluid velocity, chasing vortices on a computer seems like an attractive proposition. Küchemann[33], an aeronautical engineer, aptly characterized vortices as the 'muscles and sinews of fluid motion'. Several computational techniques have been devised to handle vortex lines or rods[34–36], sheets[37], rings[38] and so on. These attempts are very suggestive, and certainly mimic turbulence. But there are difficulties in a rational treatment of viscosity, and the highly contorted geometry of vortex structures in turbulent flows requires tools for handling that we do not yet possess.

So the future for doing turbulence on computers is likely to lie in two directions. For modest, and modestly increasing, Reynolds numbers, direct numerical solutions will provide insights into the structure of turbulent flows that will begin to surpass what is available from experiment in standard or classical flows. Experiments will continue to remain important at high Reynolds numbers, and, at lower Reynolds numbers, will act as scouts exploring unusual flow situations where surprises may await us, as they have done in the past. Engineers will chase vortices or develop hybrid techniques like large-eddy simulations, which will tackle full-scale or nearly full-scale Reynolds numbers by doing dynamics on large scales and models for small scales, the latter drawing upon all the insights that have resulted from much inspired fundamental research. These hybrid simulations will probably replace the numerous models in current use some day, but exactly when is not entirely clear, and that may well depend on how rapidly technology will provide greater computing power at low cost.

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